New Method for Measuring Properties of Nonhomogeneous Materials by a Two-Polarization Forward-Scattering Measurement

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Abstract—**A new method for measuring properties of granular materials with a two-polarization scattering measurement combined with a free-space phase measurement is introduced. The theoretical background of the measurement method is presented. Laboratory measurements of the depolarized scattering cross sections of white rice, green lentils, polystyrene beads, polyethylene beads, and mixtures of them at 10 and 35 GHz with a vector network analyzer and two horn antennas are presented. Results are compared with those based on first-order multiple scattering theory. Laboratory tests of the measurement of the relative effective permittivity of different materials by a phase measurement are presented. Also, laboratory tests of the compensation of the effects of changes in the number density and the permittivity of the inclusions of mixtures using a phase measurement are presented. A low-cost measurement setup for industrial measurements is also suggested.**

Introduction

Microwave scattering has scarcely been utilized in industrial process measurements, although many potential applications exist. For example, size and moisture distributions and density of inclusions of granular materials can be measured. This could be applied in the wood processing industry (size distribution of particles in particle board manufacturing), in energy production (particle density measurement and unburned coal detection in coal combustors), in the agricultural and food processing industries (moisture distribution of soybeans, germination grade of malt in malt kilns, density and moisture of tobacco or rice), and in the construction industry (moisture measurements of building materials). A few applications have been presented. For example, measuring methods based on microwave scattering have been suggested for measurements in coal combustors [1]. A method for correcting errors in attenuation measurements due to large particle in building materials has been discussed [2].

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The Method

The main principle of the measurement method is the measurement of the ratio of the power of the forward-scattered depolarized wave to that of the copolarized wave propagated through random media consisting of discrete scatterers. In isotropic medium, extinction (the attenuation due to absorption and scattering) is the same for both polarizations and does not affect the power ratio, which is a function of the average forward-scattering cross section and the path length of the wave in random media. The method is especially suitable for particle size distribution measurements if the frequency is chosen so that the average size of the inclusions of the material is much smaller than the wavelength (the Rayleigh approximation). Since the scattering cross section also depends on the number density, permittivity, and shape of the inclusions, in some cases the effect of changes in them should be compensated by also measuring the phase change of the copolarized wave. The phase change depends on the real part of the effective permittivity of the layer. The effective permittivity is a function of the number density, shape, and permittivity of the inclusions of the layer material, but does not depend on the size of the inclusions. The effective permittivity can be calculated with a mixing formula.

Theoretical Background

Consider the situation presented in Fig. 1. A layer of granular material is placed between two narrow beam antennas which are face to face. The spacing between the antennas is r and the thickness of the layer is d. A linearly polarized wave is transmitted through the layer from the transmitter antenna. The power ratio between cross- and copolarized waves received by the receiver antenna can be derived as [3]

$$\frac{P_\perp}{P_\parallel} = \frac{\Omega d \langle \sigma_\perp \rangle}{\pi(1 + d/r)} \tag{1}$$

where $\langle \sigma_\perp \rangle$ represents the average value of the cross-polarized forward-scattering cross section of the scattering layer and the solid angle $\Omega = \int_{4\pi} F \, d\Omega$, where F is the

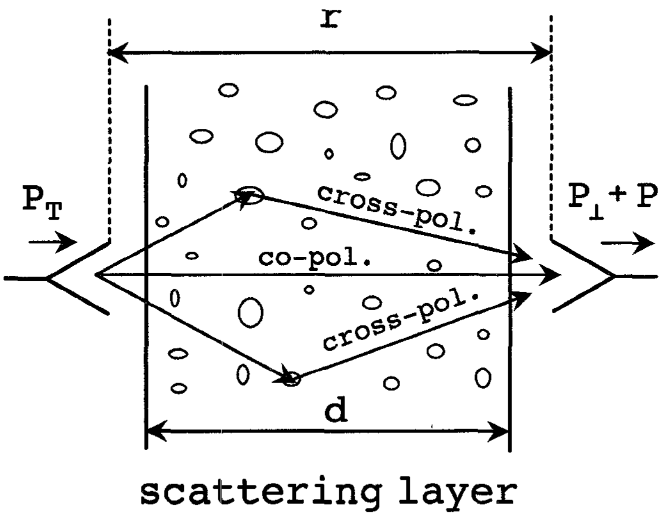

Fig. 1. The basis for (1).

normalized radiation pattern of an antenna. The power ratio depends on the gain and separation of the antennas, average cross-polarized scattering cross section of scatterers, and the thickness of the scattering layer.

The scattering cross section of a layer of randomly oriented and positioned small ellipsoidal scatterers can be calculated by applying the first-order multiple scattering theory [4] within the Rayleigh approximation. In the first-order multiple scattering theory, the scattering of each scatterer is assumed to be independent, but the wave incident on a scatterer has been attenuated by scattering and absorption along the path, and the scattered wave has likewise been attenuated by scattering and absorption before it reaches the receiver. The theory can be easily applied to other shapes of inclusions (spheres, disks, needles) by changing the depolarization factors of the ellipsoids.

If the measured material fulfills the assumptions of the first-order multiple scattering theory in the Rayleigh region, the average cross-polarized bistatic forward-scattering cross section can be calculated [5]:

$$\langle \sigma_\perp \rangle = \frac{nv^2 k^4}{4\pi} \langle |(\epsilon_r - 1)C|^2 \rangle \tag{2}$$

where n is the number of scatterers per unit volume, v the average volume of the scatterers, k the wavenumber in the host material, ϵ_r the relative permittivity of the scatterers, and C is a factor depending on the depolarization factors of an ellipsoid and the position of its axes relative to the field vector of the incident field. The angular brackets represent the ensemble average over the angles of the axes. In the case of general ellipsoids (all axes of the ellipsoid different in length), (2) becomes quite complicated, and a closed-form solution cannot be found. In [3], a closed-form solution for spheroids (ellipsoids of revolution) for practical calculations is presented.

In the case of a distribution of different sizes of ellipsoids, the average scattering cross section can be obtained [4]:

$$\langle \sigma_\perp \rangle = \int_0^\infty \sigma_\perp(D) p(D)\, dD \tag{3}$$

where $p(D)$ is the probability density function of a typical dimension D of an ellipsoid. In case of mixtures of several materials, the total scattering cross section can be calculated $\langle \sigma_\perp \rangle = \Sigma_i \langle \sigma_\perp \rangle_i$.

For (1) and (2) to be valid, the following conditions should be satisfied.

1) The average effective particle diameter is much smaller than the wavelength.
2) The distribution of the particles is tenuous.
3) The optical thickness of the layer is small.
4) The particles are randomly positioned.
5) The beams of the antennas are narrow.
6) The radiation patterns of the scatterers are isotropic.
7) The reflection coefficients of the surfaces of the layer are small.

Most of the above assumptions can be fulfilled by choosing the measurement frequency correctly and using high-gain antennas.

The change of the phase $\Delta\phi$ of the coherent wave in the layer compared to the situation without the layer can be calculated [7]:

$$\Delta\phi = (\sqrt{\epsilon_r'} - 1) k\, d \tag{4}$$

where ϵ_r' is the real part of the relative permittivity of the layer. Equation (4) can be directly used to calculate the relative permittivity of the layer from the change of phase if the phase change is monotonic ($\Delta\phi < 2\pi$). If this is not the case, a two-frequency measurement can be used, and the relative permittivity can be calculated:

$$\epsilon_r' = \left[\frac{\Delta(\Delta\phi)}{\Delta f} \frac{c_o}{2\pi\, d} + 1 \right]^2 \tag{5}$$

where δf is the difference between the measurement frequencies, $\Delta(\Delta\phi)$ is the difference between the phase changes at frequencies f and $f + \Delta f$, and c_o is the velocity of light in air. The measurement frequencies should chosen so that $\Delta(\Delta\phi) < 2\pi$.

Usually, (4) and (5) are valid if the multiple reflections in the layer are small. This is the case of the attenuation of the layer is over 10 dB. A more accurate equation for the change of the phase can be calculated using, e.g., a signal flow graph. In practice, the effect of reflections on the phase measurement is less serious than in the attenuation measurement [7].

In the case of a nonhomogeneous material, the permittivity should be replaced by the real part of the effective permittivity ϵ_{rm}, which can be calculated with, for example, the Polder–van Santen formula for n types of inclusions [8]:

$$\epsilon_{rm} = \epsilon_{rh} + \frac{s_1}{s_2}$$

$$s_1 = \frac{1}{3}\sum_{j=1}^{n} f_j(\epsilon_{rj} - \epsilon_{rh}) \sum_{i=1}^{3} \frac{\epsilon_{rm} + N_{ji}(\epsilon_{rh} - \epsilon_{rm})}{\epsilon_{rm} + N_{ji}(\epsilon_{rj} - \epsilon_{rm})}$$

$$s_2 = 1 - \frac{1}{3}\sum_{j=1}^{n} f_j(\epsilon_{rj} - \epsilon_{rh}) \sum_{i=1}^{3} \frac{N_{ji}}{\epsilon_{rm} + N_{ji}(\epsilon_{rj} - \epsilon_{rm})} \tag{6}$$

where ϵ_{rh}, ϵ_{rj} are the complex relative permittivities of the host material and jth type of inclusions, and N_{ji} and f_j are the depolarization factors and the filling factors of the jth type of inclusions.

Consider a situation where the average size of the inclusions of a layer of a granular material with a known thickness is to be measured by the two-polarized forward-scattering method. First, a calibration measurement should be done, where the cross-polarized scattering cross section of a well-known sample of the material is measured. The average size, the number density, the shape, and the relative complex permittivity of the inclusions of the sample should be known. During the actual scattering cross-section measurement, the compensation of the effects of changes in the permittivity or the number density of the inclusions of the layer to its average cross-polarized scattering cross section can be done by the following procedure. First, the phase change of the copolarized wave propagated through the layer compared to the situation without the layer is measured. Then, the effective relative permittivity of the layer is calculated with (5). The number density or the average permittivity of the inclusions of the layer is calculated with (6). This value is used to calculate the scattering cross section with (2). The calculated scattering cross section is then compared to the one calculated on the basis of the known values of the permittivity, the number density and depolarization factors of the calibration sample, and the difference between them is used to correct the measured scattering cross sections. The corrected scattering cross sections are then compared to the one measured with the calibration sample to find the change in the average size of the inclusions.

Laboratory Measurements

In laboratory tests, the scattering cross-section measurement by a two-polarization forward-scattering method, the measurement of the relative effective permittivity of granular materials with a phase measurement, and the compensation of the effects of the changes in the number density and the permittivity of the inclusions of the measured materials were tested.

The setup for laboratory measurements in shown in Fig. 2. The scatterers were placed in a styrofoam container

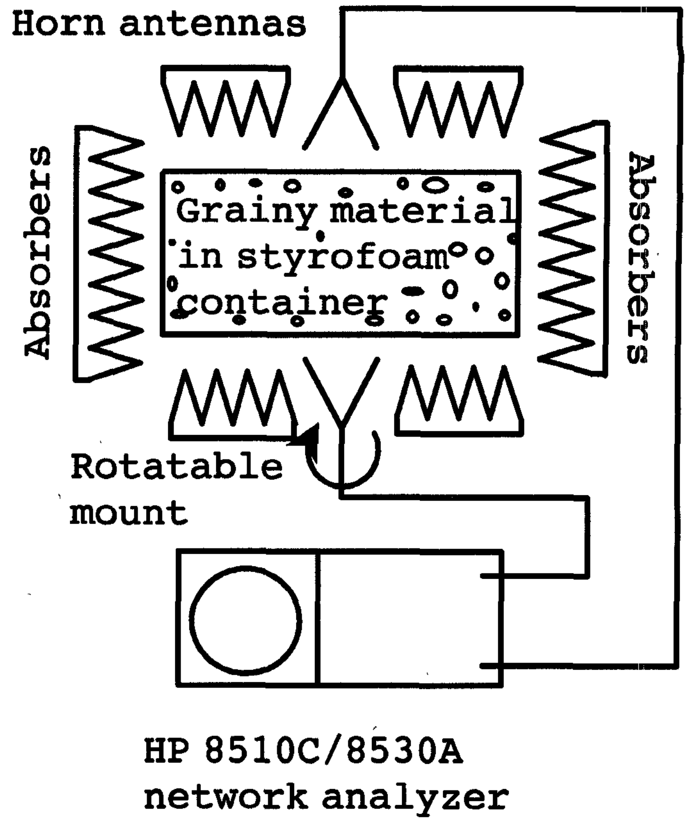

Fig. 2. The measurement setup for laboratory measurements.

between two pyramidal horn antennas (gains 21 and 18 dB and return losses less than −18 dB at 10 GHz) or two corrugated horn antennas (gains 21 dB and return losses less than −20 dB at 35 GHz). The power ratio was measured with a Hewlett-Packard 8510C/8530A network analyzer. One of the antennas could be rotated about its axis. The power ratios without scatterers were −50 dB at 10 GHz and −65 dB at 35 GHz. The measured materials at 10 GHz were dried green lentils (seeds of the leguminous crop), a mixture of green lentils and almost spherical polyethylene beads, and a mixture of white rice (wet-basis moisture content about 15%) and spherical polystyrene beads with approximately 20 wt % of rice. At 35 GHz, white rice and a mixture of white rice and polystyrene beads were measured. The average bulk densities and the complex relative permittivities of the measured materials are presented in Table I. The permittivities of lentils and rice were measured at 7.5 GHz using the cavity perturbation method with a cylindrical TE_{011}-cavity resonator. At 7.5 GHz, the real part of the measured materials ϵ_r' can be assumed to be the same as at 10 GHz. The imaginary part of the permittivity ϵ_r'' can change more as a function of frequency, but since $\epsilon_r' >> \epsilon_r''$, the effect of the changes in the imaginary part of the permittivity on the scattering cross section are negligible.

The permittivity of rice at 35 GHz was roughly approximated on the basis of the work done in [6]. The thickness

TABLE I
THE AVERAGE BULK DENSITIES ρ AND THE COMPLEX RELATIVE PERMITTIVITIES ϵ_r', ϵ_r'' OF THE MEASURED MATERIALS

Material	ρ [kg/m^3]	ϵ_r'	ϵ_r''
Lentils	940	4.7 ± 0.5	0.4 ± 0.02
Rice	1560	5.0 ± 0.3	0.3 ± 0.05
Polystyrene	1100	2.3	0.0009
Polyethylene	940	2.3	0.00058

TABLE II
THE MINIMUM VOLUMES v_{min}, MAXIMUM VOLUMES v_{max}, AVERAGE VOLUMES v, ECCENTRICITIES e, AND NUMBER DENSITIES n OF THE INCLUSIONS OF THE MEASURED MATERIALS

Material	v_{min} [10^{-8} m^3]	v_{max} [10^{-8} m^3]	v [10^{-8} m^3]	e	n [10^7 m^{-3}]
Lentils	4.7	9.4	7.7	2.1	1
Rice	1.1	1.6	1.3	0.9	4.7
Polystyrene	0.05	0.14	0.09	0.33	86
Polyethylene	2.9	3.8	3.4	0.33	2.0

of the scattering layer was varied (d = 100–150 mm at 10 GHz and d = 10–30 mm at 35 GHz) and the scattering cross sections were obtained with (1). Theoretical scattering cross sections were calculated with (2). The lentils were modeled as oblate spheroids, the rice beads as prolate spheroids, and polystyrene and polyethylene beads as spheres. The volume ranges, average volumes, eccentricities, and number densities of the inclusions of the measured materials are presented in Table II. The eccentricity is $e = \sqrt{1 - (b/a)^2}$ for prolate spheroids ($a > b = c$) and $e = \sqrt{(a/c)^2 - 1}$ for oblate spheroids ($a = b > c$), where a, b, and c are the lengths of the axes of the spheroid. The measured and theoretical bistatic forward scattering cross sections of a 1-cm-thick layer of the measured materials at 10 and 35 GHz are presented in Table III. The measured results show fairly good agreement with the theoretical ones.

The effect of the number density of the particles on the scattering cross section was tested using the measurement setup in Fig. 2 at 35 GHz with a mixture of white rice and polystyrene grains. The filling factor of rice was varied from $f = 0.17$ to $f = 0.61$. The measured scattering cross sections were compared with the theoretical ones calculated with (2). The measured and theoretical values are presented in Fig. 3. The error bars represent the scatter of the results because of the variation of the spatial distribution of the rice grains and the variation of the thickness of the layer between the samples. The scatter can be reduced to some extent by more careful preparation of the samples and increasing the number of the measurements.

The measurement of the real part of the relative effective permittivity by a phase measurement was performed with the measurement setup in Fig. 2. The measured materials were lentils, a mixture of lentils and polyethylene

TABLE III
MEASURED AND THEORETICAL CROSS-POLARIZED FORWARD-SCATTERING CROSS SECTIONS $\langle \sigma_\perp \rangle$ FOR A 1-cm-THICK LAYER OF LENTILS AND MIXTURE OF WHITE RICE AND POLYSTYRENE GRAINS (20 wt% OF RICE) AT f = 10 GHz AND FOR WHITE RICE AT f = 35 GHz

Material	f[GHz]	$\langle \sigma_\perp \rangle$ [dB] Meas.	$\langle \sigma_\perp \rangle$ [dB] Calc.
Lentils	10	−23.1 ± 2.4	−23.9 ± 2.4
Rice and polystyr.	10	−38.4 ± 3.4	−36.9 ± 1.3
Rice	35	−7.9 ± 1.0	−7.6 ± 3.0

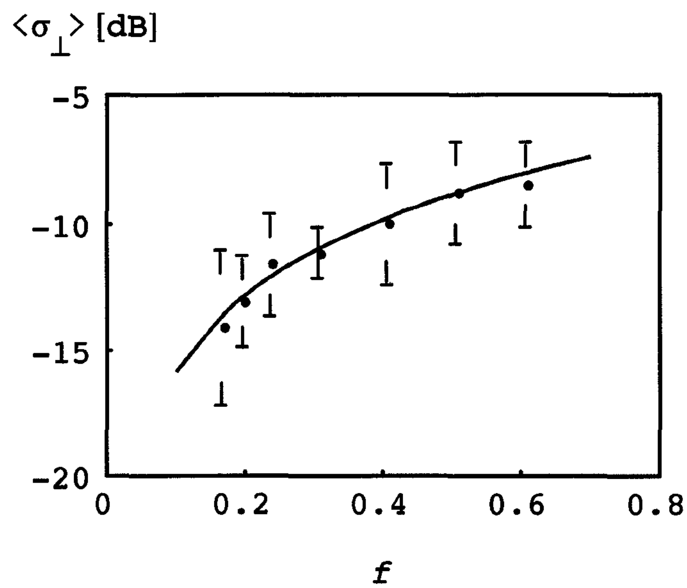

Fig. 3. The measured (●) and theoretical (solid line) cross-polarized forward-scattering cross sections $\langle \sigma_\perp \rangle$ for a 1-cm-thick layer of a mixture of rice and polystyrene grains at 35 GHz as a function of filling factor f of rice. The error bars represent the scatter of the results because of the nonhomogeneity of the samples (typically 20 measurements at each value of number density).

TABLE IV
MEASURED AND THEORETICAL REAL PARTS OF THE RELATIVE EFFECTIVE PERMITTIVITIES ϵ'_{rm} FOR LENTILS, A MIXTURE OF LENTILS AND POLYETHYLENE BEADS, AND A MIXTURE OF RICE AND POLYSTYRENE BEADS. THE MEASUREMENT FREQUENCIES WERE 9.5 AND 10.5 GHz

Material	ϵ'_{rm} Meas.	Calc.
Lentils	3.46 ± 0.2	3.50
Lentils and polyeth.	2.52 ± 0.1	2.32
Rice and polystyr.	1.83 ± 0.1	1.80

grains, and a mixture of rice and polystyrene grains. The phase change in the layer was measured at 9.5 and 10.5 GHz, and the relative effective permittivity was calculated using (5). The theoretical relative effective permittivities were calculated with (6). The measured and theoretical real parts of the relative effective permittivities for the lentils, for the mixture of lentils and polyethylene beads, and for the mixture of rice and polystyrene beads are presented in Table IV.

The compensation of the effect of changes in the number density of the inclusions was tested by measuring several mixtures of green lentils and polyethylene beads. The filling factor [$f = nv$, (2)] of the lentils was varied from $f = 0.77$ (only lentils) to $f = 0.15$. The phase change was measured with the measurement setup shown in Fig. 2. The effective permittivity of the layer was calculated with (5), and the filling factor of lentils was calculated from the phase change with (6). The scattering cross section of the layer was measured as shown before. The measurement frequencies for the phase measurement were 9.5 and 10.5 GHz, and for the scattering cross section measurement, 10 GHz. The measured values of the scattering cross section were corrected with the calculated filling factor and compared to the value measured for $f = 0.77$, which was $\langle \sigma_\perp \rangle_{0.77} = -23.4 \pm 2.0$ dB for a 1-cm-thick layer. The measured and corrected values are shown in Fig. 4. The error bars represent the scatter of the results, which is mainly due to the nonhomogeneity of the samples. The average value of the corrected scattering cross sections was $\langle \sigma_\perp \rangle_{corr} = -22.9 \pm 0.05$ dB.

The compensation of the effect of changes in the permittivity of the inclusions was tested by increasing the wet-basis moisture content of the lentils about 25 wt% units, which caused the relative permittivity of the lentils to increase from $\epsilon_r = 4.1 - j0.6$ to $\epsilon_r = 8.0 - j3.7$. The measured cross-polarized scattering cross section for a 1-cm-thick layer of dry lentils was $\langle \sigma_\perp \rangle_{dry} = -23.1 \pm 2.2$ dB. The measured value for the wet lentils was $\langle \sigma_\perp \rangle_{wet} = -17.8 \pm 1.0$ dB, and the corrected value was $\langle \sigma_\perp \rangle_{corr} = -24.3 \pm 1.0$ dB.

The measurements show that the two-polarization forward-scattering method can be used to measure properties of granular materials, and the effect of the changes in the number density and the permittivity of the inclusions on the scattering cross section of the measured material can be compensated by using a phase measurement. The scatter of the measured cross-polarized scattering cross sections is mainly due to the nonhomogeneity of the measured samples, especially in the case of thin layers of mixtures of two materials.

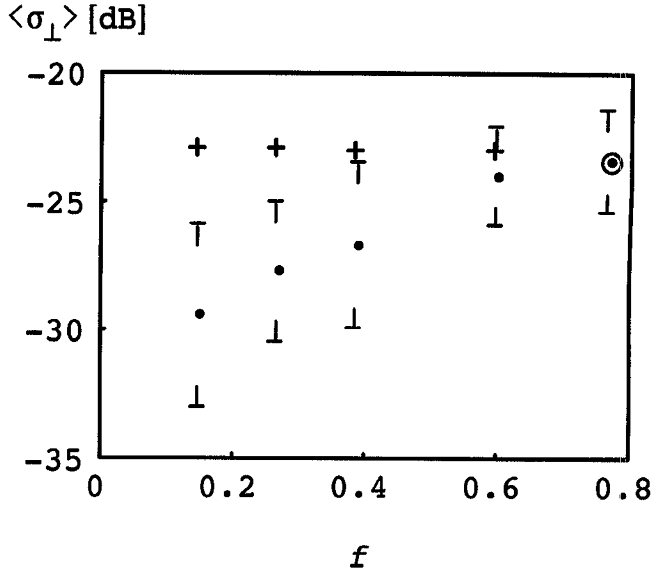

Fig. 4. The measured (●) and the corrected (+) cross-polarized bistatic forward scattering cross sections $\langle \sigma_\perp \rangle$ for a 1-cm-thick layer of a mixture of green lentils and polyethylene grains at 10 GHz as a function of the filling factor f of the lentils. The encircled marker represents the value measured for a layer containing only lentils ($f = 0.77$). The error bars represent the scatter of the results due to the nonhomogeneity of the samples (typically 20 measurements at each value of number density).

THE PRACTICAL MEASUREMENT SETUP

The suggested low-cost measurement setup for industrial measurements is presented in Fig. 5. The measuring method suggested in this paper is combined with the measurement of phase change and total attenuation of the layer. The setup consists of a swept-frequency oscillator (swept frequency for the phase measurement), two horn antennas with low cross-polarization levels (corrugated or Potter horns), an orthomode transducer, and a scalar analyzer, which can be connected to the process control system. In the case of a layer with low attenuation, the signal source of the scalar analyzer can be used and the oscillator is not needed. The operating frequency should be chosen considering the optical thickness of the layer and the average size of the inclusions. In most practical cases, a frequency between 2.5–10 GHz should be usable because at that frequency range, the particles of the most common measured materials can be assumed to be in the Rayleigh region and the size of the measurement equipment is reasonable.

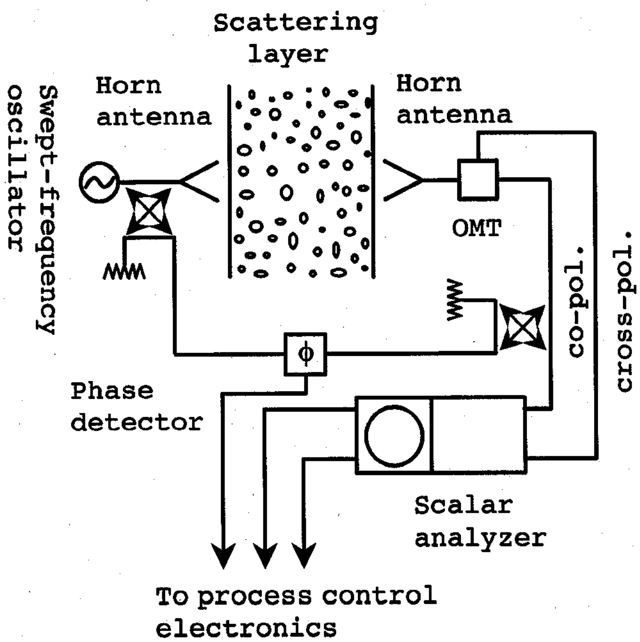

Fig. 5. The measurement setup for industrial measurements.

Conclusions

A new measurement method for measuring properties of granular materials has been presented. The method is based on a two-polarization forward-scattering measurement which is combined with a free-space phase measurement. Laboratory tests show promising results. The method seems suitable for particle size distribution measurements.

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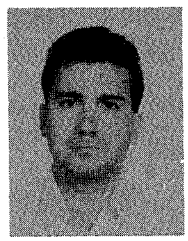

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